Fan or Propeller Blade Loads & Root Stress

1.0 Introduction

A calculation of the forces generated by a fan blade at given speed.

We are concerned with the blade root failure mode, and will determine the blade root stress for various load cases and sum as appropriate.

Loads cases considered are:

- Centripetal
- Torque reaction
- Thrust force

A finite element analysis (FEA) was conducted to determine the effect of geometric factors assymetric loads and aerodynamic loads.

The resulting stres values are compared to the published material properties, and to failure criteria based on both CAA-BHSR requirements and fatigue life considerations.

2.0 References

Petersen: Stress Concentration Factors

Multiwing: PAG properties and fan performance data

BASF Corp: Effects of time and temperature conditions on the tensile-tensile fatifue behaviour of short fibre reinforced polyamides.

3.0 Definitions

Define some units

$$rpm := \frac{2\cdot\pi}{60\cdot s}$$

$$C := K$$

$$MPa := Pa\cdot 10^{6}$$

4.0 Input data

Typical data for a 5z multiwing in a 900mm duct

$V_{tip.rec} \equiv 110 \cdot \frac{m}{s}$	Manufacturers maximum tp speed
$V_{tip.HCGB} \equiv 168 \cdot \frac{m}{s}$	HCGB maximum tp speed
$P_{engine} := 32 \cdot hp$	Power consumed by fan
$F_{thrust} := 150 \cdot lbf$	Thrust generated by the fan
$n_{blade} := 6$	Number of blades
$r_{cg} := 0.23 \cdot m$	Radius to C of G of blade
$r_{cp} := 0.35 \cdot m$	Radius to centre of pressure on blade
$r_{tip} \equiv 450 \cdot mm$	Tip radius
$r_{hub} := 100 \cdot mm$	Radius of hub
$m_{blade} := 420 \cdot gm$	Mass of blade (less root)
$OD_{root} := 39.5 \cdot mm$	Diameter of blade root
$ID_{root} := 25 \cdot mm$	
$\omega := 500 \cdot rpm, 1000 \cdot rpm .. 4000 \cdot rpm$	Fan speed range to plot in graphical output
$\omega_1 := 3000 \cdot rpm$	Fan speed for displaying results

5.0 Material Data

Glass Reinforced Polyamide (PAG)

Glass Reinforced Polyamide (PAG)
Temperature range: -40°C to +120°C Please observe penalty factors for temperatures above 40°C

Mechanical properties (Dry as moulded):	
Tensile strength:	165 Mpa
Izod impact strength notched (at 23°C):	11.0 kJ/m^2
Izod impact strength notched (at -30°C):	9.0 kJ/m^2
Tensile modulus:	8.2 Gpa

Mechanical properties (50% relative humidity):	
Tensile strength:	100 Mpa
Izod impact strength notched (at 23°C):	17.0 kJ/m^2
Izod impact strength notched (at -30°C):	9.0 kJ/m^2
Tensile modulus:	5.6 Gpa

The values for the mechanical properties are mean values and can be subject to variations due to the use of different suppliers.

Source : http://uk.multi-wing.com/Products/FanMaterials/PAG

Ultimate tensile strength

$\sigma_{UTS} := 100 \cdot MPa$

This is mean UTS. Actual values will vary and a scatter factor is required to account for variation

$Scatter_{UTS} := 0.9$

$\sigma_{UTS.material.min} := \sigma_{UTS} \cdot Scatter_{UTS}$ $\qquad \sigma_{UTS.material.min} = 90\,MPa$

$\sigma_{UTS.material.max} := \sigma_{UTS} \cdot \left(2 - Scatter_{UTS}\right)$ $\qquad \sigma_{UTS.material.max} = 110\,MPa$

Design Stress

From CAA BHSR

$\sigma_{design.ultimate.CAA}(\omega) := \dfrac{\sigma_{UTS.material.min}}{3}$ $\qquad \sigma_{design.ultimate.CAA}(\omega_1) = 30\,MPa$

Fatigue strength

$m_{fat} := 4.60 \cdot MPa$

$B_{fat} := 66.7 \cdot MPa$

Ref Effects of time and temperature conditions on the tensile-tensile fatifue behaviour of short fibre reinforced polyamides. BASF Corporation

$N_{fail} := 1 \cdot 10^7$

10^7 cycles is commonly accepted as representing infinite life

$\sigma_{fatigue} := 2 \cdot \left(B_{fat} - m_{fat} \cdot \log\left(N_{fail}\right)\right)$ $\qquad \sigma_{fatigue} = 69\,MPa$

Max stress for infinie life

$Scatter_{fatigue} := 1.0$

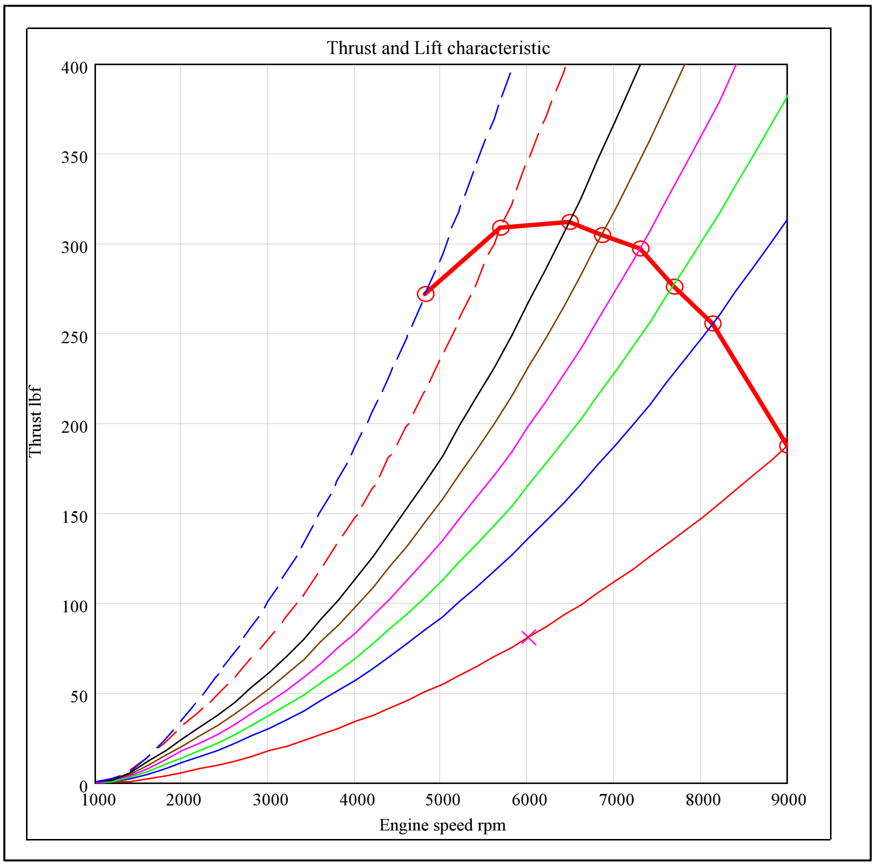
$\sigma_{design.fatigue}(\omega) := \sigma_{fatigue} \cdot Scatter_{fatigue}$ $\qquad \sigma_{design.fatigue}(\omega_1) = 69\,MPa$

No factors are required since the fatigue data is based on concentrated stress, and will be compared against the calculated concentrated stress

Thrust - Speed data

$k_p := 2.0$ Redrive ratio

Derived from multiwing data program Optimiser Rev 5

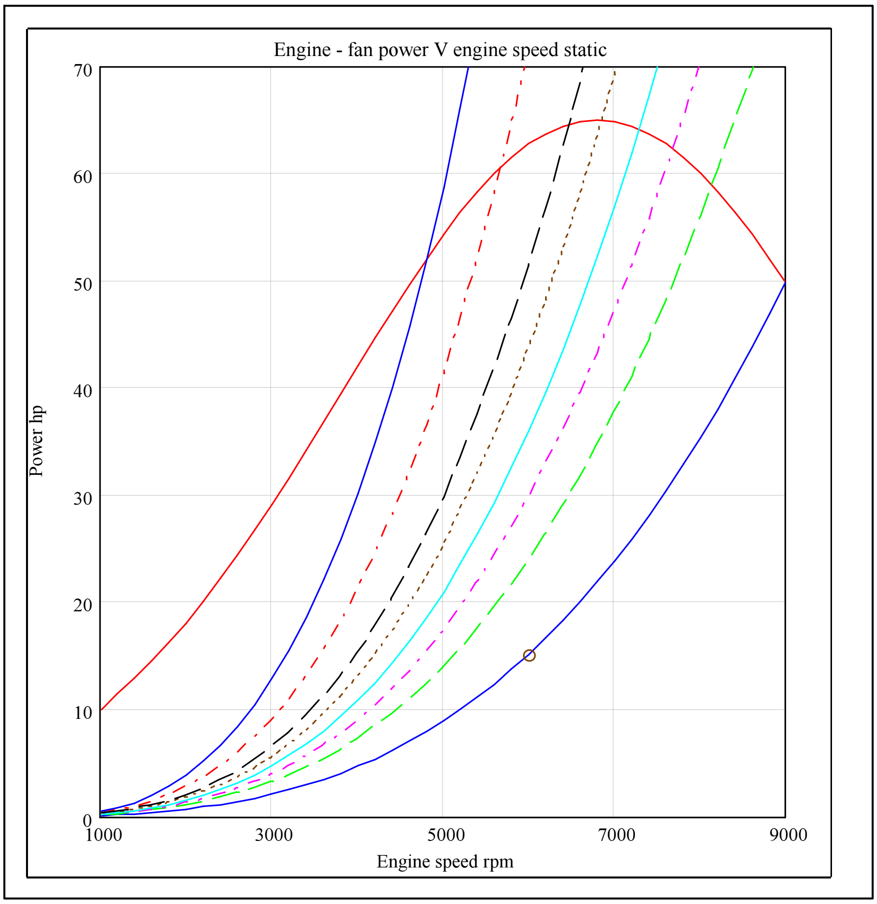


The above data for 40 deg pitch is described by:

$$T_{thrust}(\omega) := \left[-12.256 + 7.448\cdot10^{-3}\cdot\frac{\omega}{rpm}\cdot k_p + 5.07\cdot10^{-6}\cdot\left(\frac{\omega}{rpm}\cdot k_p\right)^2 + 2.375\cdot10^{-10}\cdot\left(\frac{\omega}{rpm}\cdot k_p\right)^3\right]\cdot lbf$$

$$T_{thrust}(3000\cdot rpm) = 1184.3\,N$$

Power speed data



The above data for 40 degree pitch (dotted black line) is described by

$$P_{fan}(\omega) := \left[-0.063 - 4.642 \cdot 10^{-5} \cdot \frac{\omega}{rpm} \cdot k_p + 9.252 \cdot 10^{-8} \cdot \left(\frac{\omega}{rpm} \cdot k_p \right)^2 + 2.267 \cdot 10^{-10} \cdot \left(\frac{\omega}{rpm} \cdot k_p \right)^3 \right] \cdot hp$$

6.0 Calcs

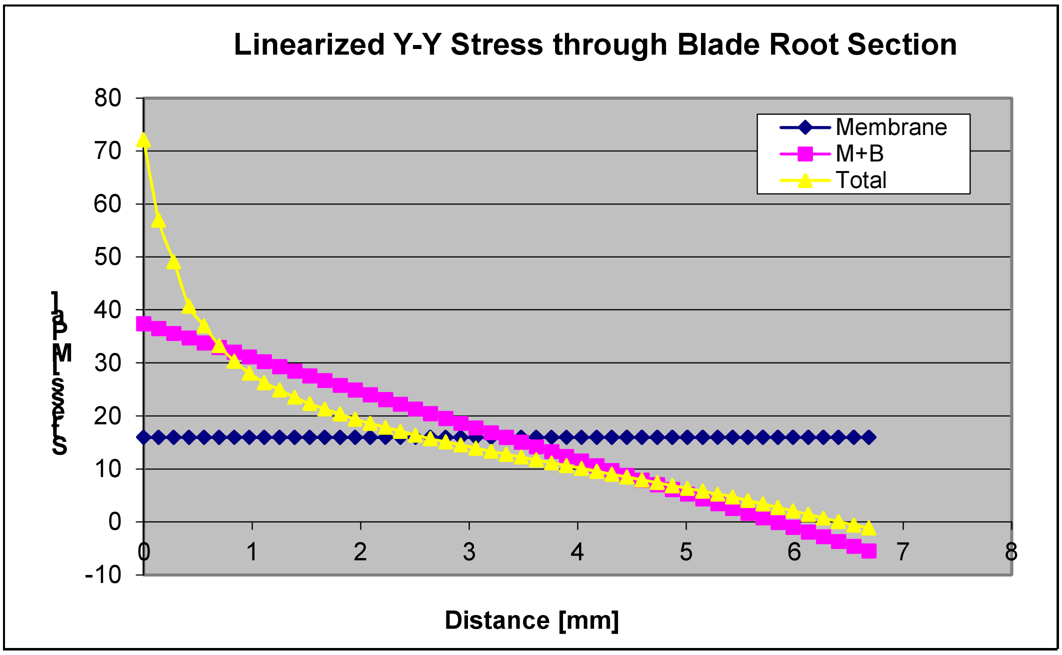
Mean Centripetal Load & Stress

$F_{centripetal}(\omega) := m_{blade} \cdot r_{cg} \cdot \omega^2$ $\qquad$ $F_{centripetal}(\omega_1) = 9534\,N$

Direct load on root section

$A_{root} := \frac{\pi}{4} \cdot \left(OD_{root}^2 - ID_{root}^2\right)$ $\qquad$ $A_{root} = 734.5\,mm^2$

A finite element anaysis was conducted on the blade root section, including a sufficient section of the blade to properly input the CF load. This shows that the CF load is not balanced, and there is a significant bending stress at the root due to the unbalance, as shown in the figure below. This results in a bending factor tha must be taken into account in the mean stress



$k_{t.CF.offset} := 2.35$

$\sigma_{centripetal}(\omega) := \frac{F_{centripetal}(\omega)}{A_{root}} \cdot k_{t.CF.offset}$ $\qquad$ $\sigma_{centripetal}(\omega_1) = 30.5\,MPa$

Bending moment & stress at root due to torque reaction

Torque on each blade due to prime mover, due to aerodynamic drag on blade

$$M_{drag}(\omega) := \frac{P_{fan}(\omega)}{\omega \cdot n_{blade}}$$ $$M_{drag}(\omega_1) = 20.6\,N\cdot m$$

Direct aerodynamic drag force on blade

$$F_{drag}(\omega) := \frac{M_{drag}(\omega)}{r_{cp}}$$ $$F_{drag}(\omega_1) = 58.7\,N$$

Bending moment on blade root due to drag force on blade

$$M_{drag}(\omega) := F_{drag}(\omega)\cdot\left(r_{cp} - r_{hub}\right)$$ $$M_{drag}(\omega_1) = 14.7\,N\cdot m$$

$$I_{root} := \pi\cdot\frac{\left(OD_{root}^{\ 4} - ID_{root}^{\ 4}\right)}{64}$$ Second moment of area for root

$$\sigma_{bending.drag}(\omega) := \frac{M_{drag}(\omega)\cdot OD_{root}}{2\cdot I_{root}}$$ $$\sigma_{bending.drag}(\omega_1) = 2.9\,MPa$$

Bending stress at root due to thrust load

Bending moment on blade hub due to thrust force on each blade

$$M_{thrust}(\omega) := \frac{T_{thrust}(\omega)}{n_{blade}}\cdot\left(r_{cp} - r_{hub}\right)$$ $$M_{thrust}(\omega_1) = 49.3\,N\cdot m$$

$$\sigma_{bending.thrust}(\omega) := \frac{M_{thrust}(\omega)\cdot OD_{root}}{2\cdot I_{root}}$$ $$\sigma_{bending.thrust}(\omega_1) = 9.7\,MPa$$

Direct aerodynamic thrust force on blade

$$F_{thrust}(\omega) := \frac{M_{thrust}(\omega)}{\left(r_{cp} - r_{hub}\right)}$$ $$F_{thrust}(\omega_1) = 197.4\,N$$

Total mean stress

The two aerodynamic bending stresses act at 90 degrees and so are vector summed to determine the overall aerodynamic stress. The overall aerodynamic stress is summed with the direct centripetal load to determine the nominal stress for comparison with the UTS.

$$\sigma_{aero}(\omega) := \sqrt{\sigma_{bending.thrust}(\omega)^2 + \sigma_{bending.drag}(\omega)^2}$$

$$\sigma_{aero}(\omega_1) = 10.1\,\text{MPa}$$

$$\sigma_{mean}(\omega) := \sigma_{aero}(\omega) + \sigma_{centripetal}(\omega)$$

$$\sigma_{mean}(\omega_1) = 40.6\,\text{MPa}$$

Stress concentration factors

Deteremined from charts available for a bar for a shoulder fillet in tension and bending.

D= 50.5mm, d= 39mm, r = 1mm

D/d = 1.295 r/d = 0.051

$k_{t.bending} := 2.02$ Ref. Peterson's Stress Concentration Factors, 3rd edition, Chart 3.10

$k_{t.direct} := 2.14$ Ref. Peterson's Stress Concentration Factors, 3rd edition, Chart 3.4

Total concentrated stress

$$\sigma_{peak}(\omega) := \left(\sqrt{\sigma_{bending.thrust}(\omega)^2 + \sigma_{bending.drag}(\omega)^2}\right)\cdot k_{t.bending} + \sigma_{centripetal}(\omega)\cdot k_{t.direct}$$

$$\sigma_{peak}(\omega_1) = 85.7\,\text{MPa}$$

Mean section stress

Calculation of the mean section stress at the various speed limits

Manufacturers recommended stress

$$\omega_{rec} := \frac{V_{tip.rec}}{r_{tip}} \qquad \omega_{rec} = 2334.3 \text{ rpm}$$

$$\sigma_{manf.rec} := \sigma_{mean}\left(\omega_{rec}\right)$$

HCGB maximum tip speed

$$\omega_{HCGB} := \frac{V_{tip.HCGB}}{r_{tip}}$$

$$\sigma_{HCGB} := \sigma_{mean}\left(\omega_{HCGB}\right)$$

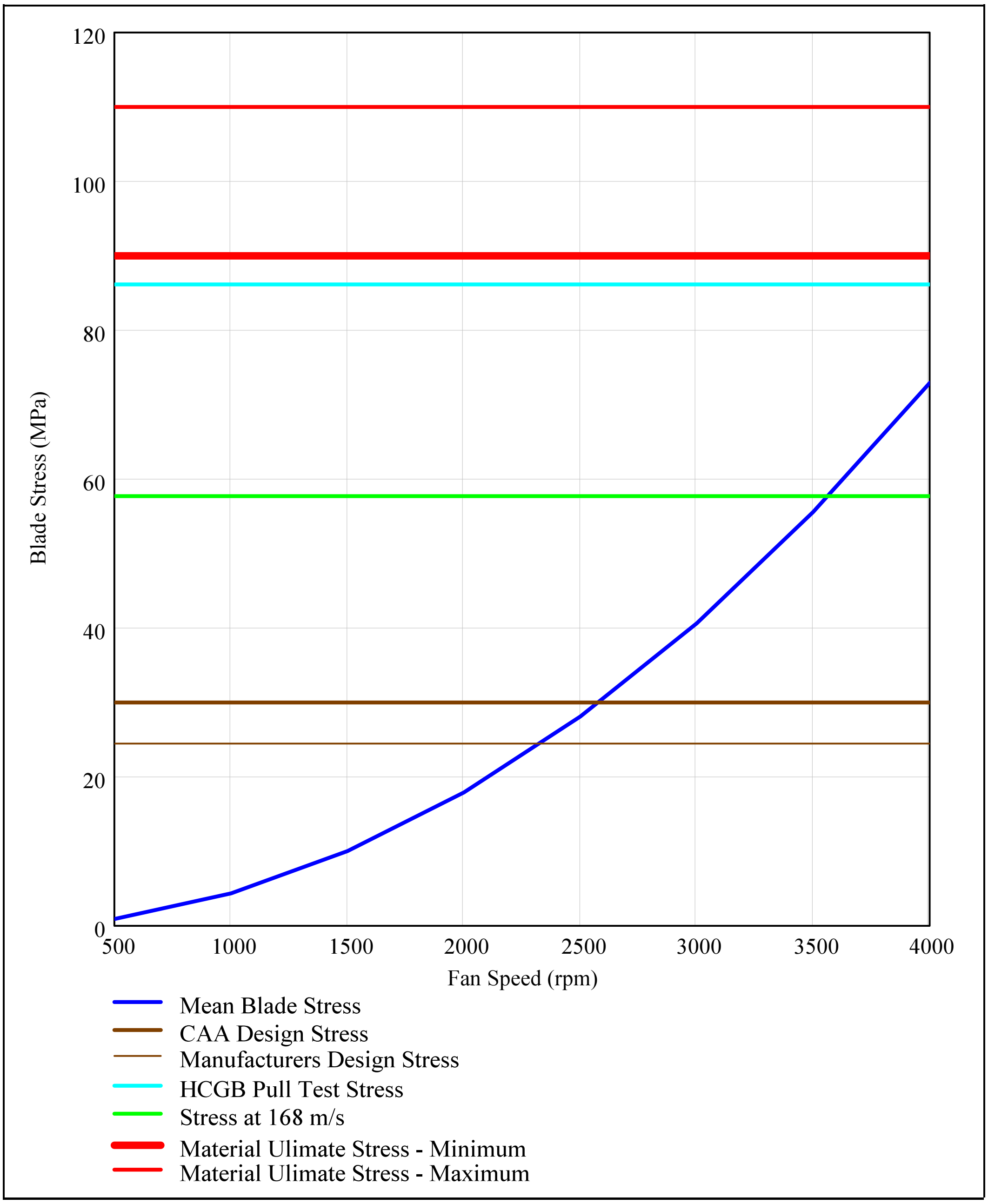
HCGB Pull test stress

The HCGB pull test operates at a load of 200% of the load at the HCGB limit

$$\sigma_{HCGB.Pull.Test} := 2 \cdot \sigma_{centripetal}\left(\omega_{HCGB}\right)$$

Mean section stress plotted against fan speed, with various limits plotted

Calculations based on mean section stress. These will always look conservative because the method is not accounting for fatigue, creep, material scatter, shock loads etc, which have to be covered by the crude use of large design factors.



$\sigma_{UTS.material.min} = 90\,MPa$ — UTS at minimum material properties

$\sigma_{design.ultimate.CAA}(\omega_1) = 30\,MPa$ — Maximum stress allowable per BHSR reqiurements

$\sigma_{mean}(\omega_{HCGB}) = 57.7\,MPa$ — Root stress at 168 m/s tip speed

$\sigma_{mean}(\omega_{rec}) = 24.5\,MPa$ — Root stress at 110 m/s tip speed

$\sigma_{mean}\left(\frac{122\cdot m\cdot s^{-1}}{r_{tip}}\right) = 30.2\,MPa$ — Root stress at 122 m/s tip speed

Peak fillet stress - Fatigue design method

Calculation of the peak fillet concentrated stress at the various speed limits

Manufacturers recommended stress

$$\omega_{rec} := \frac{V_{tip.rec}}{r_{tip}} \qquad \omega_{rec} = 2334.3\ \text{rpm}$$

$$\sigma_{manf.rec.peak} := \sigma_{peak}\left(\omega_{rec}\right)$$

HCGB maximum tip speed

$$\omega_{HCGB} := \frac{V_{tip.HCGB}}{r_{tip}}$$

$$\sigma_{HCGB.peak} := \sigma_{peak}\left(\omega_{HCGB}\right)$$

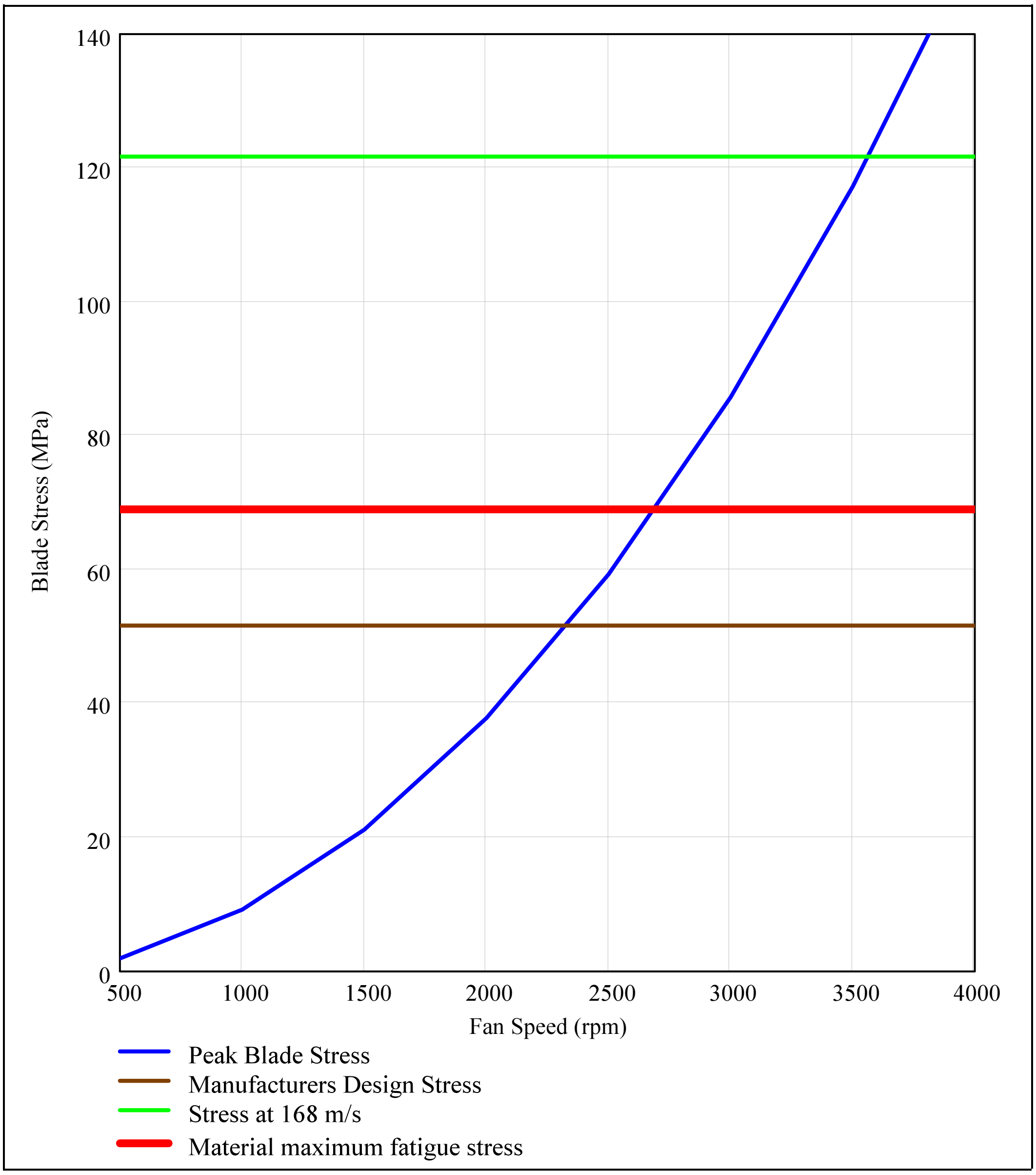
HCGB Pull test stress

The HCGB pull test operates at a load of 200% of the load at the HCGB limit

$$\sigma_{HCGB.Pull.Test.peak} := 2\cdot\sigma_{centripetal}\left(\omega_{HCGB}\right)\cdot k_{t.direct}$$

Mean section stress plotted against fan speed, with various limits plotted

Calculations based on mean section stress. These will always look conservative because the method is not accounting for fatigue, creep, material scatter, shock loads etc, which have to be covered by the crude use of large design factors.



$\sigma_{design.fatigue}(\omega_1) = 69\,\text{MPa}$ — Maximum peak concentrated stress for 10 fatigue cycles

$\sigma_{peak}(\omega_{HCGB}) = 121.7\,\text{MPa}$ — Root stress at 168 m/s tip speed

$\sigma_{peak}(\omega_{rec}) = 51.6\,\text{MPa}$ — Root stress at 110 m/s tip speed

$\sigma_{peak}\left(\frac{127\cdot m\cdot s^{-1}}{r_{tip}}\right) = 69\,\text{MPa}$ — Root stress at 127m/s tip speed

7.0 Summary

A typical 900mm 6 bladed 5z fan was selected to calculate the maximum fan speed based on material stress. Material strength data was obtained for both static strength and fatigue strength.

A calculation was completed for the chosen fan using two failure criteria:

- Ultimate strength design criteria
- Fatigue strength design criteria

Ultimate strength design criteria used mean section stress excepting that it was noted via FEA that the CF load was causing significant bending in the critical section, because the load is not centrally located over the area, leading to significant bending stress. Since the bending effects a significant volume of material it needs to be accounted for in the UTS design calculation, therefore a factor was determined from the FEA and applied to the calculation of mean section stress.

The mean section stress including the CF related bending was compared to various proposed design stress limits as described:

- CAA Design Stress. Calculated by dividing the min material UTS by 3 per the CAA recomendation in BHSR.
- Manufacturers Design Stress. Calculated mean section stress at the maximum design speed nominated by the manufacturer
- HCGB Pull Test Stress. Calculated stress when a 2x centripetal load is applied
- Stress at 168 m/s. Stress calculated for the nominated tip speed.

The fatigue strength design criteria used peak fillet stress. This was calculated from the mean section stress multiplied by a concenration factor. The concentration factors were obtained from a standard text (Petersen) and were verified by FEA.

UTS Design Criteria - Discussion of Results

The maximum speed allowed by the manufacturer is 2330rpm (110 m/s). This compares to a maximum speed calculated according to the CAA Design Stress of 2600 rpm (122 m/s).

The blade root stress at a an speed of 168 m/s is shown, and may be compared to the root stress during the standard pull test. The pull test does not result in 2x the stress found in the blade at 168 m/s, due to the effect of aerodynamic bending loads during operation that are not taken into account in the test.

It is noted that the blade stress developed at 168 m/s is approximately 2x the CAA limit, but still retains some design margin when compared to the minimum UTS.

Fatigue Design Criteria

The maximum speed allowed by the manufacturer is 2330 m/s as before, and may be compared to the maximum speed calculated for a fatigue life of 10^7 cycles which is 2695 rpm (127 m/s).

Overall

The UTS Design Criteria adopted by the BHSR (CAA) and the Fatigue Design Criteria agree remarkably well for infinite fatigue life, in suggesting tip speed limit in the region of 122 to 127 m/s. This is a little higher than the manufacturers maximum recommended speed of 110 m/s but less than the historically adopted limit of 168 m/s.

A tip speed of 168 m/s remains within the material minimum UTS, but with significantly reduced safety margin. The stress at 168 m/s (122 MPa) is found to be exceed the fatigue criteria for 10^3 cycles (106 MPa). This suggests that a fan run at 168 m/s will be accumulating low cycle fatigue damage at an appreciable rate, and will operate safely for significantly less than 1000 cycles. It is not possible to estimate the fatigue life more accurately since no low cycle fatigue data is available.

Conclusion

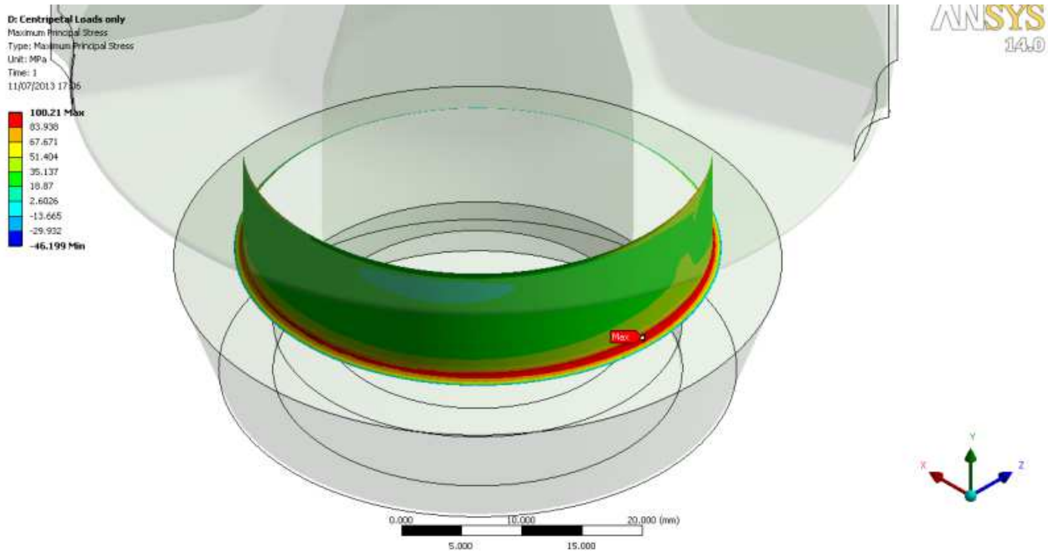
A maximum tip speed of 168 m/s may suitable for operation with a limited fatigue life and additional controls to mitgate the risk of failure.

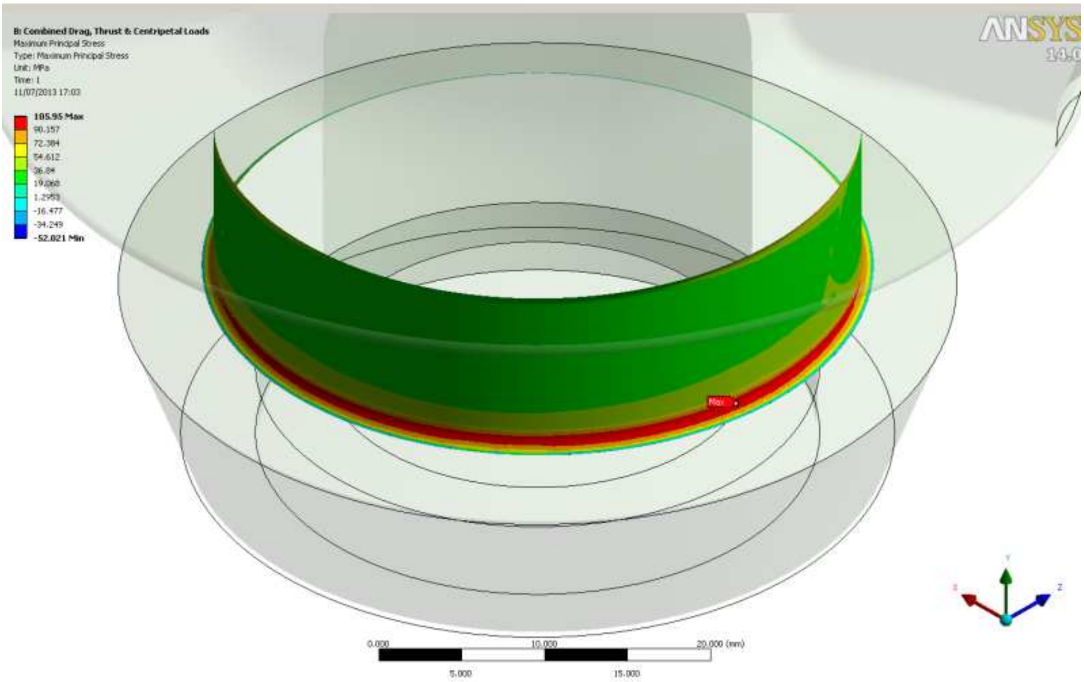
A maximum tip speed of 127 m/s is suitable for operation with unlimited fatigue life.

The BHSR design criteria is recommended for the calculation of maximum tip speeds. It is noted that offset centripetal loads, aerodynamic loads and geometric factors will need to be taken into account in such calculations.

The calculation presented here may be used for other fan blades of the same material with a reasonable degree of geometric similarity. Where there is no geometric similarity, a FEA may be needed to determine the offset & geometric factors

8.0 Appendix - FEA Model





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